DETERMINATION OF THE COEFFICIENTS OF LINEAR EXPANSION OF FIBERGLAS BY THE UNSTEADY-STATE METHOD

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Inzhenerno-Fizicheskii Zhurnal, Vol. 13, No. 1, pp. 37–43, 1967

UDC 536.37

This paper describes a modification of the previously proposed method of determining the coefficients of linear expansion of solids [1] and also gives the results of an investigation of the thermal expansion of fiberglasses. Their thermal expansion shows pronounced anisotropy.

The relationship between the length of a body and its temperature $l = f(t)$ in the absence of phase changes is given by the formula

$$l = l_0 + \alpha l_0 t + \beta l_0 t^2, \qquad (1)$$

where α and β are parameters which are constant for the particular material and the particular relationship. The actual value of the coefficient of linear expansion at a particular temperature is given by the expression

$$\alpha_t = \frac{1}{l_0}\frac{dl}{dt} = \alpha + 2\beta t. \qquad (2)$$

Formula (1) assumes a uniform temperature distribution throughout the body, which is equivalent to assuming that the body is in a steady-state, uniform temperature field. When a body changes from one temperature to another the concept of body temperature becomes indefinite, since the uniform temperature distribution throughout the body is upset. In this case we cannot speak of the temperature of the body without first stipulating the method of averaging the temperatures which the body has at different points at the particular instant.

It is known that during free cooling or heating the length of a body changes monotonically. Hence, at any instant during cooling the measured length of the body will correspond to some temperature at which it would have the same length in a uniform temperature field. We will call this temperature the temperature of the body in the unsteady state. We now find in the cooling specimen a position for a thermocouple junction at which the measured temperature is equal to the temperature of the body.

If a heated specimen of investigated material in the form of a thin solid cylinder or block is enclosed in a sufficiently good lateral heat-insulating shell, leaving the end faces exposed, temperature stratification will be set up along the specimen when it cools. There will be practically no temperature gradient in cross sections of the specimen. In this case the mean bulk temperature Θ_V of the specimen will be equal to its mean linear temperature Θ_l:

$$\Theta_V = \Theta_l = \int \frac{\Theta dV}{V} = \int_0^{l/2} \Theta\, dz \Big/ \frac{l}{2},$$

where $\Theta = t - \vartheta = f(z)$. The origin of the length scale coincides with one of the end faces of the specimen.

If we assume that the temperature is distributed in the specimen in accordance with the law $\Theta = \gamma z$, where γ is a proportionality factor, then

$$\Theta_V = \gamma \int_0^{l/2} z dz \Big/ \frac{l}{2} = \gamma \frac{l}{4}.$$

It follows from the condition of equality of the measured temperature and the mean bulk temperature ($\Theta_V = \Theta$) that the position of the thermocouple will be determined by the following value of z:

$$z = l/4 = 0.25l.$$

If we assume that $\Theta = \eta z^2$, where η is a proportionality factor, then

$$\Theta_V = \eta \int_0^{l/2} z^2 dz \Big/ \frac{l}{2} = \eta \frac{l^2}{12}.$$

Table 1

Results of Observations and Calculation of Coefficients of Ebonite Specimen (l = 107 mm, room temperature ϑ = 20° C, galvanometer division δ = 0.165 deg/div)

No. of observation	L	N	Δt	No. of observation	L	N	Δt
1	95.0	96.7	15.96	15	25.0	54.6	9.01
2	90.0	93.6	15.44	16	20.0	50.7	8.36
3	85.0	90.6	14.95	17	15.0	47.3	7.80
4	80.0	88.9	14.64	18	10.0	45.5	7.51
5	75.0	84.6	13.96	19	5.0	41.2	6.80
6	70.0	81.5	13.45	20	0.0	37.6	6.20
7	65.0	78.5	12.95	21	−5.0	34.3	5.66
8	60.0	75.5	12.46	22	−10.0	31.2	5.15
9	55.0	72.5	11.96	23	−15.0	26.1	4.31
10	50.0	69.4	11.45	24	−20.0	24.3	4.01
11	45.0	65.5	10.81	25	−25.0	21.2	3.50
12	40.0	63.1	10.41	26	−30.0	17.6	2.90
13	35.0	60.0	9.90	27	−35.0	14.0	2.31
14	30.0	57.0	9.40	28	−40.0	11.1	1.83

Table 2

Observations Selected from Graph of $L = f(\Delta t)$ for Calculation of Coefficients α and β ($L_{\vartheta} = -56.5\ \mu$)

Number of observation	Δl $L-L_{\vartheta}$	Δt	Number of observation	Δl $L-L_{\vartheta}$	Δt
1	151.5	15.96	19	61.5	6.80
2	146.5	15.44	20	56.5	6.20
3	141.5	14.95	21	51.5	5.66
5	131.5	13.96	22	46.5	5.15
6	126.5	13.45	24	34.5	4.01
7	121.5	12.95	25	31.5	3.50
8	116.5	12.46	26	26.5	2.90
9	111.5	11.96	27	21.5	2.31
Mean I	130.9	13.89	Mean II	41.5	4.57

$(\Delta l/\Delta t)_{I} = 9.42$ $(\Delta l/\Delta t)_{II} = 9.08$

Table 3

Results of Measurements of Coefficients of Linear Expansion of Some Fiberglas Specimens

Description of specimen	Direction in which coefficient was measured	Temperature range, °C, in which observations were made	Coefficients of linear expansion		
			$\alpha \cdot 10^6$	$\beta \cdot 10^6$	$\alpha_{20} \cdot 10^6$
$\varepsilon=0°$, PN-1k resin, T_1 glass cloth	perpendicular to layer	18—60	95.2	0.87	130
$\varepsilon=0°$, PN-1k, T_1	parallel to layer	20—60	15.1	0.00	15.1
$\varepsilon=0°$, PN-1k, T_1	" " "	20—65	25.2	0.00	25.2
$\varepsilon=90°$, PN-1k, T_1	" " "	19—65	21.0	0.00	21.0
$\varepsilon=90°$, PN-1k, T_1	perpendicular to layer	18—50	180.8	0.48	200
Resin	immaterial	20—40	182.8	0.18	190
$\varepsilon=90°$, NSP-205 resin, T_1	parallel to layer	16—46	11.1	0.00	11.1
$\varepsilon=90°$, NSP-205 resin, T_1	perpendicular to layer	15—28	277.4	0.44	295
$\varepsilon=0°$, NSP-205 resin, T_1	along warp parallel to layer	16—43	4.7	0.00	4.7
$\varepsilon=0°$, NSP-205 resin, T_1	along warp perpendicular to layer	16—40	170.0	1.90	246
$\varepsilon=0°$, NSP-205 resin, T_1	along weft parallel to layer	18—48	24.1	0.00	24.1
$\varepsilon=0°$, NSP-205 resin, T_1	along weft perpendicular to layer	16—50	218.0	0.90	254

In this case

$$z=\frac{l}{\sqrt{12}}=0.29\,l.$$

Thus, the position of the thermocouple junction in the specimen will be in the range $0.25l < z < 0.29l$. As the most probable value of z we can take the mean value:

$$z=0.27l.$$

Hence, if formula (1) is to be valid for any instant during cooling, the temperature measured at sections

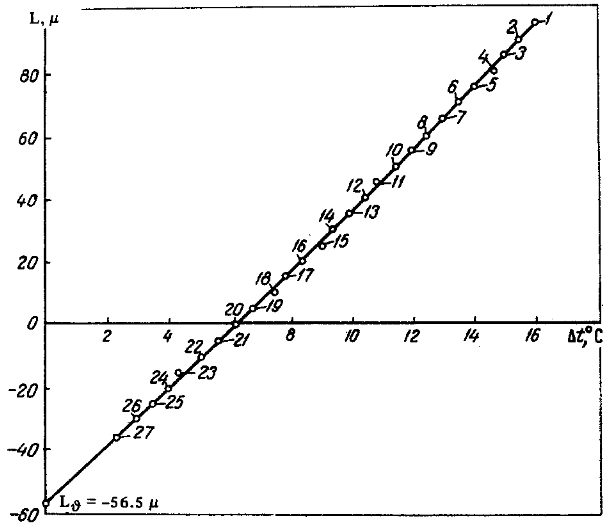

Fig. 1. Graph of relationship between optimeter readings and galvanometer readings $L = f(\Delta t)$.

at a distance of $0.27l$ from the end faces of the specimen must be used. For measurement of the temperature in this case it is convenient to use a differential thermocouple with one junction fitted in a small recess made in the specimen at the indicated distance. The other junction is placed in a medium with a constant temperature ϑ approximately equal to the temperature of the room in which the specimen is cooling. The circuit of the differential thermocouple includes a galvanometer graduated in degrees with a 100-degree scale. The changes in length of the specimen can be easily measured by a horizontal optimeter with quartz or glass points. If a vertical optimeter is used it is essential to ensure identical cooling conditions on the end faces of the specimen. This can be done by means of an additional quartz support mounted on the optimeter stage. The specimen without its insulating case, but with the thermocouple attached to it, is first held until it is completely heated to the high or low temperature. It is then enclosed in its case and mounted on the optimeter. The changes in length and temperature must be recorded simultaneously. Formulas for determining the parameters α and β of relationship (1) can be obtained in the following way.

Applying formula (1) to different instants during cooling of the specimen we obtain

$$l_i-l_\vartheta=A(t_i-\vartheta)+B(t_i^2-\vartheta^2),$$

where

$$A=\alpha\,l_0;\quad B=\beta\,l_0.$$

Putting $l_i - l_\vartheta = \Delta l_i$ and $t_i - \vartheta = \Delta t_i$, we will have

$$\Delta l_i/\Delta t_i = A + B(t_i+\vartheta)\ \ldots. \qquad (3)$$

Deriving average equations of form (3) for the start and end of cooling, we obtain

$$\left.\begin{aligned}\left(\frac{\Delta l}{\Delta t}\right)_{\rm I}&=A+B(t_{\rm I}+\vartheta)\\ \left(\frac{\Delta l}{\Delta t}\right)_{\rm II}&=A+B(t_{\rm II}+\vartheta)\end{aligned}\right\}\ldots, \qquad (4)$$

from which

$$B=\frac{\left(\frac{\Delta l}{\Delta t}\right)_{\rm I}-\left(\frac{\Delta l}{\Delta t}\right)_{\rm II}}{\Delta t_{\rm I}-\Delta t_{\rm II}},$$

$$A=\left(\frac{\Delta l}{\Delta t}\right)_{\rm I}-B(\Delta t_{\rm I}+2\vartheta),$$

where

$$\left(\frac{\Delta l}{\Delta t}\right)_{\rm I}=\frac{\sum_1^n \Delta l_i}{\sum_1^n \Delta t_i};\quad \left(\frac{\Delta l}{\Delta t}\right)_{\rm II}=\frac{\sum_1^k \Delta l_j}{\sum_1^k \Delta t_j};$$

$$\Delta t_{\rm I}=\frac{\sum_1^n \Delta t_i}{n};\quad \Delta t_{\rm II}=\frac{\sum_1^k \Delta t_j}{k}.$$

Knowing the values of A and B and measuring the length of the specimen $l \simeq l_0$ with a slide caliper, we obtain:

$$\alpha=\frac{A}{l};\quad \beta=\frac{B}{l}.$$

Knowing α and β, we can find from formula (2) the actual values of the coefficient of expansion for the prescribed temperatures

$$\alpha_t=\alpha+2\beta t$$

and thus determine the temperature relationship $l = f(t)$. We note that $\Delta l_i = l_i - l_\vartheta = L_i - L_\vartheta$, where L_i and L_ϑ are the length readings on the optimeter scale corresponding to readings N_i and $N_\vartheta = 0$ on the galvanometer scale, $\delta N_i = \Delta t_i$, where δ is the galvanometer scale division in degrees on the 100-degree scale. $[\delta]$ = deg/div.

To determine the reading L_ϑ and also to exclude gross errors from the observations we plot a graph of the relationship $L = f(\Delta t)$ (Fig. 1) and in the further treatment of the observations we take only those points (L_i, Δt_i) which lie closest to the most probable curve $L = f(\Delta t)$. The value of L_ϑ is given by the intersection of the graph with the L axis.

We give an example of the determination of the coefficients α and β for an ebonite specimen (Table 1), the expansion coefficients of which are known from determinations by the steady-state method: $\alpha = 71.0 \cdot 10^{-6}$ 1/deg, $\beta = 0.32 \cdot 10^{-6}$ 1/deg^2, $\alpha_\vartheta = 83.8 \cdot 10^{-6}$ 1/deg. Observations selected from the graph of $L = f(\Delta t)$ for the calculation of the coefficients α and β ($L_\vartheta = 56.5\mu$) are given in Table 2. Calculation of coefficients α, β, and α_ϑ:

$$B = \frac{(\Delta l/\Delta t)_{I} - (\Delta l/\Delta t)_{II}}{\Delta t_{I} - \Delta t_{II}} = 0.0365 \ \mu/\text{deg}^2,$$

$$A = (\Delta l/\Delta t)_{I} - B(\Delta t_{I} + 2\vartheta) = 7.45 \ \mu/\text{deg},$$

$$\alpha = \frac{A}{l} = 69.6 \cdot 10^{-6} \ 1/\text{deg},$$

$$\beta = \frac{B}{l} = 0.34 \cdot 10^{-6} \ 1/\text{deg}^2,$$

when $t = \vartheta \quad \alpha_\vartheta = \alpha + 2\beta\vartheta = 83.2 \cdot 10^{-6} \ 1/\text{deg}$.

A comparison of the results obtained by this method with the results of the steady-state method and an analysis of the accuracy of the method show that the coefficient α can be determined to within ±3% and β to within ±10%. The changes in length and temperature must be measured to within ±2%.

We have used the described method to determine the coefficients of linear expansion of fiberglas. Fiberglas, which is just as strong as metals, differ very greatly from them in thermal properties. This must be taken into account in the manufacture of articles for use in different temperature conditions. Our determination of the coefficients of thermal expansion of fiberglas and an investigation of their temperature dependence showed that:

1) they are characterized by pronounced anisotropy of thermal expansion. Thermal expansion along the fibers of glass cloth is only 1/20 to 1/10 of the expansion in the transverse direction;

2) the coefficients of linear expansion depend on the difference in the properties of the binders, the density, and mode of packing of the layers of glass cloth;

3) the coefficients of linear expansion of fiberglas can change with the passage of time owing to further polymerization of the binder and temperature hysteresis of the length [2];

4) for some fiberglas specimens the ratio of the coefficients (β/α) in the formula $l = l_0 + \alpha l_0 t + \beta l_0 t^2$, where l and l_0 are the lengths of the specimen at t and $t_0 = 0°$, respectively, according to the 100-degree scale, can be fairly high, about $\beta/\alpha \simeq 0.01$ 1/deg. Neglect of the coefficients β, which is very often done in practice, leads to gross errors in the calculation of the linear dimensions of fiberglas components. For instance, for components of length $l = 20$ cm with coefficients $\alpha_\perp = 200 \cdot 10^{-6}$ 1/deg, $\beta_\perp = 2 \cdot 10^{-6}$ 1/deg^2 perpendicular to the layer, and $\alpha_\parallel = 10 \cdot 10^{-6}$ 1/deg, $\beta_\parallel = 0 \cdot 10^{-6}$ 1/deg^2 parallel to the layer, the error due to neglect of β in the case of a temperature change from 0 to 50° C will be 1 mm. In fact,

$$l - (l_0 + \alpha l_0 t) = \beta l_0 t^2 =$$
$$= 2 \cdot 10^{-6} \cdot 20 \cdot 50^2 = 0.1 \text{ cm} = 1 \text{mm}.$$

If the anisotropy is neglected the error in this example will be 3 mm:

$$\Delta l_\perp = l - l_0 = \alpha_\perp l_0 t + \beta_\perp l_0 t^2 = 200 \cdot 10^{-6} \cdot 20 \cdot 50 +$$
$$+ 20 \cdot 10^{-6} \cdot 20 \cdot 50^2 = 0.3 \text{ cm} = 3 \text{ mm},$$
$$\Delta l_\parallel = \alpha_\parallel l_0 t + \beta_\parallel l_0 t^2 = 10 \cdot 10^{-6} \cdot 20 \cdot 50 + 0 =$$
$$= 0.01 \text{ cm} = 0.1 \text{ mm}, \quad \Delta l_\perp - \Delta l_\parallel \simeq 3 \text{ mm}.$$

Thus, it is clear that in calculating the linear dimensions of fiberglas articles the anisotropy of the thermal expansion and the nonlinearity of the change in linear dimensions with change in temperature must be taken into account.

We determined the coefficients of linear expansion of some fiberglas specimens (Table 3).

NOTATION

ϑ is the ambient temperature; V is the volume of specimen; l is the length of specimen; z is the coordinate axis along which distances are measured.

REFERENCES

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25 October 1966

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